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# DETERMINATION OF THE ALTITUDE OF AEROPLANES.

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THE object of this paper is to add something to the discussion of a somewhat novel problem which has arisen as a result of certain developments in the art of aviation during the past year.

Up to the time of the aviation meet at Los Angeles in February, 1910, no greater height had been recorded in the flight of a "heavier than air" machine than about fifteen hundred feet. At that meet Paulhan, in a Blériot machine, reached a height of 4165 feet, the height being determined by triangulation. The reading of his aneroid was 4600 feet.

At the Montreal meeting, in June, Walter E. Brookins attained an altitude of 3523 feet, determined by triangulation, his baroscope reading 200 or 300 feet higher; and at Atlantic City, in July, the same aviator raised the record to 6175 feet, as determined by a triangulation similar to that of the first method described in the following pages. The height by barograph was 6200 feet.

When the project of an aviation meeting at Cambridge was being discussed, the writer agreed to be responsible for the accurate measure of altitudes; and the results obtained and the methods used form the subject of this paper. At that time no official accounts of the experience at other places in making such measurements had been published, but it was agreed that it would be acceptable to the aviators that they should attempt to reach their greatest altitudes when in a vertical plane passing through the starting line of the course, and as nearly as possible at a point over the grandstand. It was also agreed that only one machine should be under observation at any time. Under these conditions the arrangements now to be described were planned. The accompanying map (Plate 1) of the Squantum peninsula and the surrounding country will serve to make the situation clear. The map shows the positions of the two base lines, and gives the ground plan of the flight of Brookins on September 8.

## FIRST METHOD, — THE FIELD BASE.

A measured base of 5000 feet was established on the line NS reaching from the extreme northern limit of the course, across the field, passing above the center of the grandstand, and terminating at a point just beyond the boulevard recently built by the Metropolitan Park Commission. The north and south direction was chosen in order that the observations might not be interfered with by the sun at the times of day most favorable for flight. It was expected that the aviators would cross this line near its center on each coil of the ascending spiral, and that the greatest height attained at such a crossing would be taken as the height of record.

At each end of the field base was placed a sextant mounted on a stand (Plate 2), with its plane in the vertical plane of the base line, and the line of sight pointed toward the grandstand, which was nearly midway between the stations. A target placed in the line of sight at a sufficient distance from each sextant served as a substitute for the horizon line, and when the image of the aeroplane was brought to coincide with the target, as it crossed the plane of reference, the altitude was read directly from the vernier of the sextant. The height of the northern station was eighteen feet and of the southern station twenty-six feet above mean low water; it is sufficiently accurate to assume that the sextants were in the same horizontal plane twenty-two feet above mean low water and nine feet above the center of the field.

After some experiments the target was given the form shown in Plate 2. The central lozenge was one foot square and the extreme horizontal length four feet. The wings served to make an approximate setting as the aeroplane approached the plane from either side. The horizon mirror of the sextant was replaced by a larger mirror extending about three inches from the sextant plane, the silvering being removed from a strip one eighth of an inch wide perpendicular to the plane of the sextant. Through this opening the target could be seen by the naked eye placed somewhat to the left of the telescope, and the aeroplane could thus be picked up and followed and an approximate setting made some time before coming to the reference plane. At the critical moment the eye was placed at the telescope and the image of the aeroplane made to cross the center of the lozenge of the target. The use of a light blue shade between the index and horizon glass greatly facilitated the observation by cutting off the glare of the sky. The clamp of the index was replaced by a roller turned by a good-sized milled head, and forming a very convenient "quick slow motion." A "finding plane" of a size corresponding to the new horizon glass was attached to the index glass and carried a level perpendicular to the plane of the sextant, by means of which that plane might be made vertical. These three modifications are all plainly shown in Plate 2.

The observers were connected with each other and with the committee room at the grandstand by a telephone line devoted to this use alone. A chart, shown about one eighth size (Figure 1), was at hand from which the height corresponding to simultaneous observations of the altitude at either end of the base could be read off.

The abscissas and ordinates represent the angles $N$ and $S$ respectively, and the curves give the values of $h$ for each 100 feet, computed from the above formula. The scale was one quarter of an inch for each degree of $N^\circ$ or $S^\circ$, and on this scale it was not difficult to read with an error of less than ten feet up to altitudes of 15,000 feet.

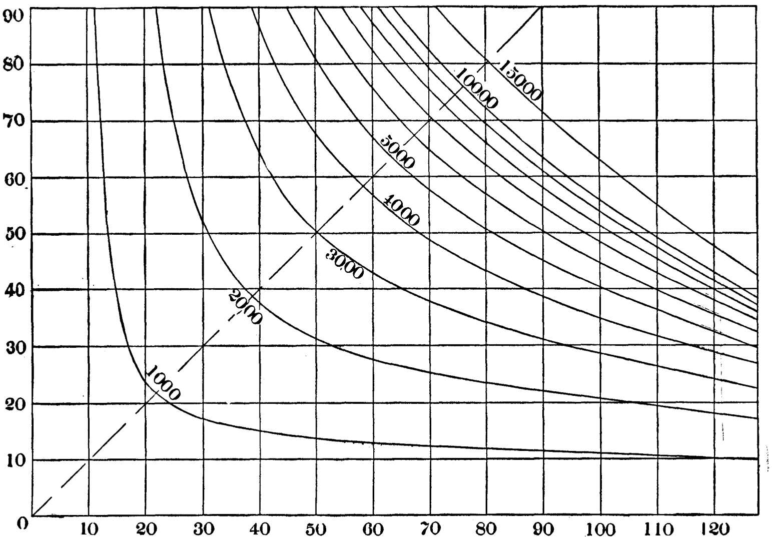


Figure 1. Chart for determining approximate altitude from sextant observations.

This chart was not used, as was expected, to give out the altitude a few seconds after each crossing, since the committee deemed it wiser not to make any announcements of altitudes till the end of the day.

## The Distant Base.

As a check on the observation at the field, and in order to give data for a closer investigation of all the circumstances of the flight, a second base was occupied lying nearly east and west, about two and a half miles south of the aviation field. Satisfactory stations were found, one on Forbes Hill in Quincy, at a height of 128 feet above the aviation field (141 feet above mean low water), and the other on the estate of Mrs. E. M. Carey, in Milton, at a height of 71 feet. The length of the base line (QM upon the map, Plate 1) was 6236 feet. The stations were visible to each other, and, although the grandstand was visible from neither, aeroplanes could be observed as soon as they had risen to the height of 50 or 75 feet.

The position of the base was selected so that observations could not at any time be interfered with by the sun, and that the angle of elevation could be measured without an eye prism up to an altitude of more than 15,000 feet above the center of the field.

The distance from the field was not so great that it was necessary to read the angles with great accuracy, while it was sufficiently great to reduce materially the difficulty in following the rapidly moving aeroplanes, so that it was possible to observe quickly and accurately both horizontal and vertical angles without a finding device.

The instruments used were a special theodolite by C. L. Berger reading horizontal angles to 10″ and vertical angles to 20″, and a Buff mining transit reading both horizontal and vertical angles to 20″. Both instruments had full vertical circles, and inverting eyepieces magnifying about thirty diameters.

Through the kindness of Mr. Carl Keller, the New England Telephone Company connected the two end stations and the central station at the field during the entire period of the meet.

It was thus possible to insure simultaneous observations at the two stations in both coördinates. That they were practically simultaneous is clearly shown by the tables which follow, in which the approximate times are given as noted at each station. Where the times differ it is almost always possible to make an exact agreement by altering one or both by a single second, so that the mean of the times would rarely be in error by that amount.

The error of the observers' watches was found by a telephone comparison with the official chronometer on the field immediately before each flight.

By combination of the observations we were able not only to compute the altitudes of the aeroplanes at intervals of forty seconds or less, but also to locate their corresponding positions as projected vertically on the ground plane, and thus to plot so many points of the spiral both in ascent and descent as to form an interesting and useful record of all the details of the flight.

It may here be remarked that the use of this method seems preferable to any that requires the aviator to attain his maximum height at a given point or line. If thus restricted, the Blériot monoplane especially is at a disadvantage, as its construction renders it extremely difficult for the operator to see the ground at points anywhere near directly beneath him. The discussion of his ascending spiral, checked by observations of his descent, fixes his highest point quite accurately without placing any burden upon the aviator himself. (In reference to this point, see the flight of Grahame-White discussed on p. 42.)

The character of the results obtained may best be shown by the records of some of the flights and the methods of discussing them which follow.

To illustrate the whole process, we may take the observations of the flight of Brookins, September 12, the last day when any considerable flight was made, and at a time when the observers had had several days of practice. No great altitude was reached, but the flight was in many respects typical, and exemplifies some of the advantages of the two methods used, as controlling and checking each other.

TABLE I.

Flight of Brookins, Sept. 12, 1910, as Measured by Sextants. Base Line, Distance between Stations, 5000 feet. Sextants 9 feet above Course.

| North Station. | | | | South Station. | | | |
|---|---|---|---|---|---|---|---|
| No. | Direction. | Time. | Vert. Angle. | Time. | Vert. Angle. | Computed. | Chart. |
| 1 | 1st, E | $5^h\ 58^m\ 30^s$ | 17° 7′ | $5^h\ 58^m\ 35^s$ | 13° 57′ | 697 ft. | 694 ft. |
| 2 | 2nd, W | | obtuse[1] | 6 0 49 | 8 56 | | |
| 3 | 2nd, E | 6 6 55 | 39 28 | 7 4 | 25 50 | 1533 | 1529 |
| 4 | 3rd, W | 10 45 | 24 4 | 10 53 | 60 26 | 1791 | 1799 |
| 5 | 3rd, E | 12 35 | 47 50 | 39 | 31 15 | 1967 | 1964 |
| 6 | 4th, W | 13 10 | 71 8 | 17 | 20 34 | 1672 | 1664 |
| 7 | 4th, E | 13 35 | 42 24 | 45 | 20 12 | 1338 | 1314 |

[1] Too large to be measured by the sextant.

If the angular altitudes at the two stations are $N°$ and $S°$ respectively, and the length of the field base 5000 feet, the altitude in feet, $h$, given in column 7, is computed by the expression

$$h = \frac{5000}{\cot N + \cot S}.$$

The values given in column 8 were read directly from the chart referred to above on page 27. A correction of 9 feet has been added to each value, as the sextant base was higher than the middle of the field by that amount.

Only four crossings were well observed in the ascent and two in the descending spiral. The second observation, however, though incomplete, served a purpose, as will appear later. Not only the altitudes, but the projections of the crossing points on the ground line are given by the observations, since the distances of the projection from the north and south stations are $h \cot N$ and $h \cot S$ respectively, and six points in the spiral are thus fixed, making, with the start and finish, eight points in all thus completely determined.

## Observations at the Distant Base.

As compared with the eight determinations made by the sextants, twenty-one simultaneous observations were made at the Quincy-Milton base. These are shown in detail in Table II. For purposes of comparison the results of the sextant observations are inserted, unnumbered but in chronological order.

The values of the altitudes in columns 5 and 9 are computed as follows:

Let $h$ be the height of the aeroplane above the center of the field,
$C$, the length of the base line, 6236 feet,
$q = 128$, and $m = 71$ feet, the heights of Q and M above the center of the field,
$\alpha$ and $\beta$, the horizontal angles between the aeroplane and base line at Q and M respectively,
$A$ and $B$, the angular altitude of the aeroplane at Q and M respectively.

Then
$$h = C \tan A \sin \beta \operatorname{cosec} (\alpha + \beta) + q,$$
$$= C \tan B \sin \alpha \operatorname{cosec} (\alpha + \beta) + m.$$

All effects due to the curvature of the earth and refraction are neglected as being less than the errors of measurement which were expected to occur. The accuracy actually attained would seem, however, to justify the application of such corrections.

## TABLE II.

FLIGHT OF BROOKINS, SEPT. 12, 1910, AS MEASURED BY TRANSITS. BASE LINE, DISTANCE BETWEEN STATIONS, 6236 FEET. COURSE 13 FEET ABOVE MEAN LOW WATER.

| QUINCY STATION Height above Course 128 ft. | | | | | MILTON STATION Height above Course 71 ft. | | | |
|---|---|---|---|---|---|---|---|---|
| No. | Time. | Vert. Angle. | Hor. Angle. | Alt. | Time. | Vert. Angle. | Hor. Angle. | Alt. |
| 1 | 5h 57m 31s | 1° 54′ | 70° 35′ | 576 ft. | 5h 57m 30s | 2° 17′ | 82° 10′ | 583ft. |
| 2 | 58 12 | 2 18 | 74 30 | 649 | 58 13 | 2 36 | 77 31 | 653 |
| | 58 32 | by sextants | | 687 | | | | |
| 3 | 58 43 | 2° 33′ | 77° 50′ | 700 | 58 40 | 2 47 | 74 19 | 705 |
| 4 | 59 24 | 2 45 | 83 3 | 769 | 59 22 | 2 51 | 70 47 | 770 |
| 5 | 6 0 20 | 2 39 | 77 59 | 861 | 6 0 20 | [2 53] | 79 15 | 865[1] |
| | 0 49 | by sextants | | 940 | | | | |
| 6 | 2 2 | 3° 24′ | 67° 53′ | 1052 | 2 00 | [3 54] | 88 29 | 1054 |
| 7 | 3 8 | 4 4 | 65 6 | 1169 | 3 10 | 4 49 | 89 42 | 1190 |
| 8 | 3 53 | 4 44 | 67 13 | 1274 | 3 51 | 5 23 | 86 4 | 1276 |
| 9 | 4 42 | 5 22 | 65 31 | 1358 | 4 42 | 6 10 | 86 7 | 1362 |
| 10 | 5 13 | 5 42 | 63 12 | 1402 | 5 13 | 6 40 | 87 35 | 1404 |
| 11 | 5 55 | 6 13 | 62 58 | 1431 | 5 53 | 7 16 | 85 43 | 1434 |
| 12 | 6 32 | 6 17 | 70 48 | 1482 | 6 32 | 6 50 | 79 19 | 1488 |
| | 7 0 | by sextants | | 1524 | | | | |
| 13 | 7 3 | 6° 1′ | 76° 40′ | 1518 | 7 2 | 6 17 | 76 1 | 1527 |
| 14 | 7 42 | 5 45 | 83 45 | 1565 | 7 41 | 5 44 | 71 44 | 1571 |
| 15 | 8 15 | 5 57 | 87 53 | 1597 | 8 15 | 5 45 | 67 55 | 1602 |
| 16 | 9 16 | 6 38 | 86 56 | 1633 | 9 15 | 6 21 | 66 47 | 1636 |
| 17 | 10 2 | 7 24 | 83 14 | 1693 | 10 0 | 7 10 | 68 4 | 1693 |
| 18 | 10 40 | 8 10 | 79 59 | 1759 | 10 38 | 8 1 | 69 10 | 1758 |
| | 10 49 | by sextants | | 1782 | | | | |
| 19 | 11 41 | 9° 14′ | 72° 12′ | 1889 | 11 40 | 9 37 | 74 10 | 1887 |
| 20 | 12 25 | 8 7 | 73 25 | 1949 | 12 23 | 8 34 | 78 2 | 1955 |
| | 12 37 | by sextants | | 1958 | | | | |
| 21 | 13 11 | 6° 4′ | 75° 58′ | 1674 | 13 10 | 6 22 | 79 8 | 1674 |
| | 13 14 | by sextants | | 1663 | | | | |
| | 13 40 | by sextants | | 1311 | | | | |
| | 14 40 | alighted | | | | | | |

[1] In the case of observations 5 and 6 the vertical angles as read off at Milton have each been decreased by 1°. This correction is justified not only by the agreement thus brought about between the two stations, but by the sextant observation at 6h 0m 49s which, as is shown on page 34, indicates an altitude of 940 feet.

To find the horizontal distances $D_q$ and $D_m$ from Q and M, we have, with little extra work and from the same data of computation,

$$D_q = C \sin \beta \operatorname{cosec} (\alpha + \beta) \quad \text{and} \quad D_m = C \sin \alpha \operatorname{cosec} (\alpha + \beta),$$

and by use of these two distances the points are easily plotted with reference to the base line. It is sufficiently accurate, however, and more convenient, to determine the points graphically by drawing lines at the proper angles with the base by means of a protractor and straight edge, and marking their intersection as the position of the aeroplane.

The computations were quickly made on mimeographed forms following the scheme shown on page 33, which gives the original computations for observations 5 and 6 of Table II, using, of course, the uncorrected data. The discrepancy of these results pointed out at once the faulty observations.

The general agreement of the values at the two stations seems to show that the height is fixed by each observation with an error of less than five feet; but observations 5 and 6 at Quincy and Milton, as computed from the recorded data, show differences too large to be ascribed to error of observation. The sextant observation at $6^h\ 0^m\ 49^s$, although the angle was too great to be measured at the north sextant station (the spiral being far out beyond the course), still shows that the observations at the Quincy station give the correct value, and there seems to be good reason for assuming that the vertical angles were read 1° too large in each case at the Milton station. It appears in the discussion of all the flights that the error most frequently made was in reading the vertical angles. In the whole series of observations but two errors are indicated in the reading of the horizontal angles, one of which will be referred to later.

## Time-Altitude Curves.

By plotting the successive values of $h$, given in Table II as abscissas, with the corresponding times as ordinates, we have the curve given below, Figure 2 (right), the slope of which gives the rate of ascent in feet per minute. The points corresponding to transit observations are indicated by circles, the sextant observations by crosses. The rate of ascent is about 100 feet per minute, and the descent nearly ten times as rapid.

## COMPUTATION OF ALTITUDE FROM THE FIFTH, AND SIXTH OBSERVATIONS OF TABLE II.

Date 1910, Sept. 12 — $C$ = base line
Aviator, Brookins — $h$ = height
$A$, $\alpha$, $q$ = alt., az. and elevation at Quincy Station.
$B$, $\beta$, $m$ = alt., az. and elevation at Milton Station.

| Time | $6^h\ 0^m\ 20^s$ | Time | $6^h\ 0^m\ 20^s$ | Time | $6^h\ 2^m\ 2^s$ | Time | $6^h\ 2^m\ 0^s$ |
|---|---|---|---|---|---|---|---|
| $A$ | 2° 39′ | $B$ | 3° 53′ | $A$ | 3° 24′ | $A$ | 4° 54′ |
| $\beta$ | 79 15 | $\alpha$ | 77 59 | $\beta$ | 88 29 | $\alpha$ | 67 53 |
| $\alpha + \beta$ | 157 14 | $\alpha + \beta$ |  | $\alpha + \beta$ | 156 22 | $\alpha + \beta$ |  |
| $\log C$ | 3.79493 | $\log C$ | 3.79493 | $\log C$ | 3.79493 | $\log C$ | 3.79493 |
| $\tan A$ | 8.66543 | $\tan B$ | 8.83175 | $\tan A$ | 8.77387 | $\tan B$ | 8.93313 |
| $\sin \beta$ | 9.99231 | $\sin \alpha$ | 9.99038 | $\sin \beta$ | 9.99985 | $\sin \alpha$ | 9.96681 |
| $\csc(\alpha + \beta)$ | 0.41231 | $\csc(\alpha + \beta)$ | 0.41231 | $\csc(\alpha + \beta)$ | 0.39698 | $\csc(\alpha + \beta)$ | 0.39698 |
| $\log(h - q)$ | 2.86498 | $\log(h - m)$ | 3.02937 | $\log(h - q)$ | 2.96563 | $\log(h - m)$ | 3.09185 |
| $h - q$ | 733 | $h - m$ | 1070 | $h - q$ | 924 | $h - m$ | 1236 |
| $q$ | 128 | $m$ | 71 | $q$ | 128 | $m$ | 71 |
| $h$ | 861 | $h$ | 1141 | $h$ | 1052 | $h$ | 1307 |

## Representation of the Spiral.

From the observations of Table II we may plot the ground plan of the spiral described in the ascent, and, with somewhat less certainty, that of the much more rapid descent. Each of the sextant observations, too, fixes a point on the spiral, although taken alone they would give little information as to the actual path of the aviator.

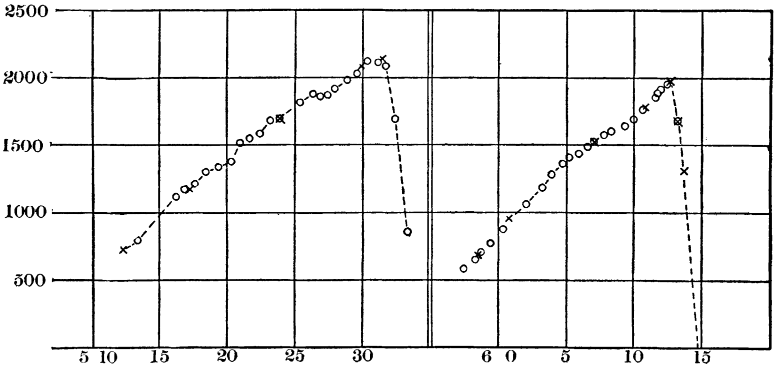


Figure 2. Time-altitude curves of Grahame-White (left) and of Brookins (right), September 12.

In Figure 3 are plotted the points as thus determined, each being numbered to correspond with the observation of Table II on which it depends. The points of crossing the base line are determined by the sextant observations of Table I.

The elevation of the path, as shown in the lower half of the figure, is determined by combining the altitudes from Table I and II with the projections of the observed points of the spiral above.

It is noteworthy that the spiral serves to give with some accuracy the height at the point of the crossing at $6^h\,0^m\,49^s$, where the angle was too great to be measured at the north end of the base. The figure shows that the projection of that point, which is marked by the dot between points 5 and 6 of the spiral, is about 900 feet north of that station, or 5900 feet from the south station, at which the vertical angle was measured as 8° 56′. This gives a height of 940 feet (assuming the elevation of the station as 13 feet), and a little further investigation shows that the angle at the north station was about 134°, and confirms the record of the observer that it was not measurable, as the sextant reads only to 128°. The error in position of the projected point is probably less than 150 feet, corresponding to an uncertainty of about 24 feet in the deduced height and 3° in the angular altitude at the north station.

In order that an intelligent judgment may be formed of the general accuracy attained, of the nature and frequency of erroneous observations, which must always occur in such a series, of the possibility of

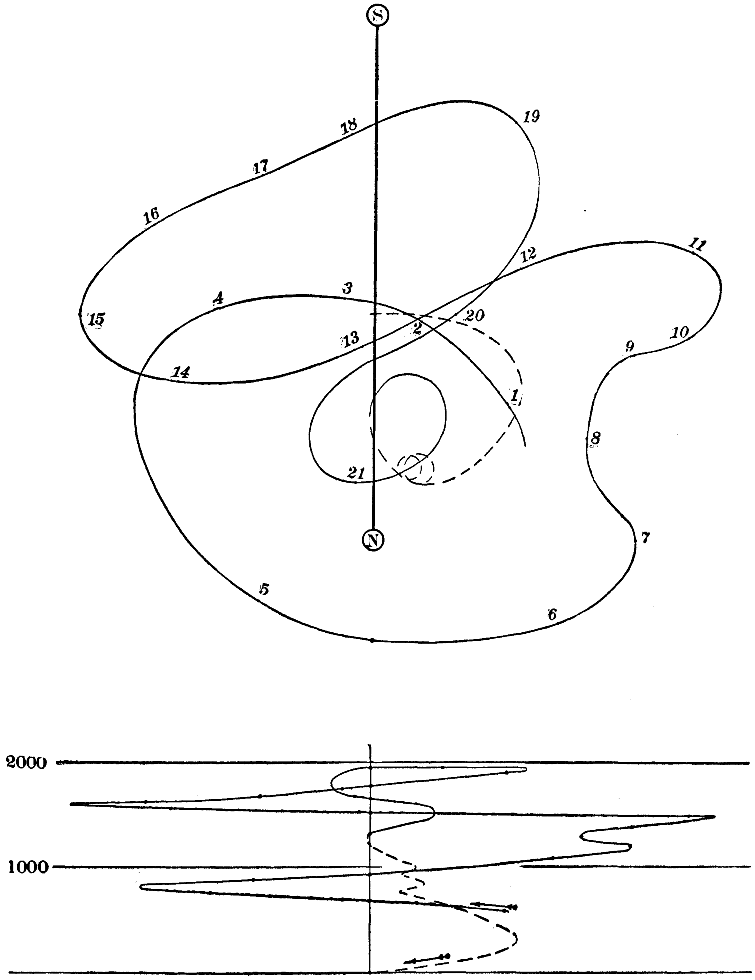


Figure 3. Spiral of Brookins, September 12, plotted from observations of Tables I and II.

correcting such observations, and the effect of rejecting them, three additional sets are given in Tables III, IV, and V. The first is that of Brookins on September 10, in which he reached the greatest altitude of the meet, with three discordant observations, one of which has been corrected, apparently with reason, by altering the vertical angle at the Quincy station by 1°. The others would be brought into line by changes of 10′ and 30′ in the vertical angles at Milton station, but no outside evidence is furnished by the sextants or spiral to justify such a change. In both cases the spiral, Figure 5, indicates that the horizontal angles were probably correctly recorded.

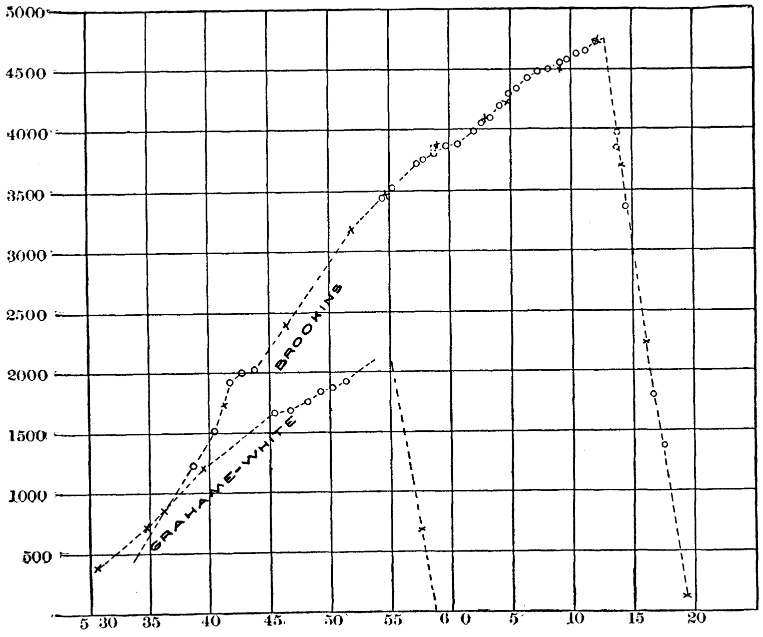


FIGURE 4. Flights of September 10: Brookins (Wright biplane), Grahame-White (Farman biplane).

The second flight, that of Brookins on September 8, shows two corrections of vertical angle and one change of the time of observation at Quincy, three vertical angles unrecorded at Milton, and one observation marked worthless, the result from which differs 17 feet from that at Quincy. This was on the second occasion of making observations, and naturally contains more errors and uncertainties, but in no case is the height uncertain by so much as five feet.

### TABLE III.

Flight of Brookins, Sept. 10, 1910. Wright Biplane. Base Line 6236 feet, Course 13 feet above Mean Low Water.

| | Quincy Station. Height above Course 128 ft. | | | | Milton Station. Height above Course 71 ft. | | | |
|---|---|---|---|---|---|---|---|---|
| No. | Time. | Vert. Angle. | Hor. Angle. | Altitude. | Time. | Vert. Angle. | Hor. Angle. | Altitude. |
| 1 | $5^h 38^m 37^s$ | 3° 54′ | 68° 59′ | 1220 ft. | $5^h 38^m 40^s$ | 4° 24′ | 88° 7′ | 1222 ft. |
| 2 | 40 22 | 5 47 | 68 28 | 1511 | 40 30 | 6 28 | 84 30 | 1518 |
| 3 | 41 40 | [7] 56 | 80 41 | 1910[1] | 41 44 | 7 55 | 71 44 | 1919 |
| 4 | 42 41 | 7 44 | 90 59 | 2005 | 42 45 | 7 8 | 64 54 | 1981 |
| 5 | 43 42 | 7 12 | 90 20 | 2019 | 43 46 | 6 51 | 67 6 | 2023 |
| 6 | 54 13 | 11 12 | 69 49 | 3432 | 54 19 | 12 10 | 88 15 | 3450 |
| 7 | 55 11 | 11 53 | 82 51 | 3514 | 55 15 | 11 49 | 75 9 | 3527 |
| 8 | 57 10 | 15 3 | 85 36 | 3713 | 57 15 | 14 19 | 68 35 | 3715 |
| 9 | 57 52 | 15 43 | 82 34 | 3753 | 57 55 | 15 11 | 70 19 | 3753 |
| 10 | 58 35 | 16 16 | 79 5 | 3803 | 58 40 | 16 16 | 72 42 | 3849 |
| 11 | 59 41 | 16 50 | 73 42 | 3858 | 59 45 | 17 20 | 76 48 | 3865 |
| 12 | 6 0 35 | 16 36 | 69 31 | 3874 | 0 40 | 17 43 | 81 7 | 3877 |
| 13 | 1 55 | 15 13 | 66 1 | 3986 | 2 0 | 10 48 | 87 55 | 3986 |
| 14 | 2 37 | 14 21 | 71 21 | 4039 | 2 36 | 15 18 | 84 41 | 4050 |
| 15 | 3 15 | 14 29 | 80 20 | 4092 | 3 20 | 14 29 | 76 24 | 4091 |
| 16 | 4 5 | 16 52 | 81 48 | 4201 | 4 10 | 16 27 | 72 0 | 4199 |
| 17 | 4 50 | 17 52 | 74 43 | 4295 | 4 55 | 18 17 | 77 13 | 4295 |
| 18 | 5 30 | 17 58 | 68 8 | 4338 | 5 34 | 19 23 | 83 22 | 4339 |
| 19 | 6 22 | 17 23 | 60 27 | 4419 | 6 26 | 20 3 | 92 31 | 4427 |
| 20 | 7 14 | 15 36 | 58 32 | 4472 | 7 19 | 18 12 | 98 5 | 4478 |
| 21 | 8 2 | 14 33 | 63 25 | 4496 | 8 6 | 16 21 | 94 55 | 4503 |
| 22 | 9 2 | 14 24 | 73 2 | 4530 | 9 7 | 15 14 | 85 42 | 4549 |
| 23 | 9 39 | 14 48 | 78 48 | 4569 | 9 42 | 15 3 | 79 47 | 4576 |
| 24 | 10 20 | 15 29 | 85 33 | 4616 | 10 25 | 15 5 | 72 52 | 4626 |
| 25 | 11 9 | 17 41 | 87 2 | 4646 | 11 14 | 16 46 | 68 45 | 4645 |
| 26 | 12 2 | 18 59 | 76 43 | 4719 | 12 6 | 19 13 | 76 17 | 4731 |
| 27 | 13 39 | 13 45 | 68 5 | 3974 | 13 42 | 14 30 | 88 33 | 3844 |
| 28 | 14 24 | 11 21 | 81 10 | 3356 | 14 28 | 11 24 | 76 40 | 3364 |
| 29 | 16 34 | 7 18 | 67 52 | 1789 | 16 38 | 8 8 | 83 35 | 1799 |
| 30 | 17 24 | 4 41 | 65 55 | 1375 | 17 28 | 5 23 | 89 54 | 1381 |

[1] Vert. angle at Q has been changed from the recorded value of 6° 56′.

The third flight, Figure 2 (left), is that of Grahame-White on September 12, immediately preceding that of Brookins. The seventh observation shows a corrected vertical angle at Milton, and the twelfth a correction of the horizontal angle at the same station, — one of two instances de-

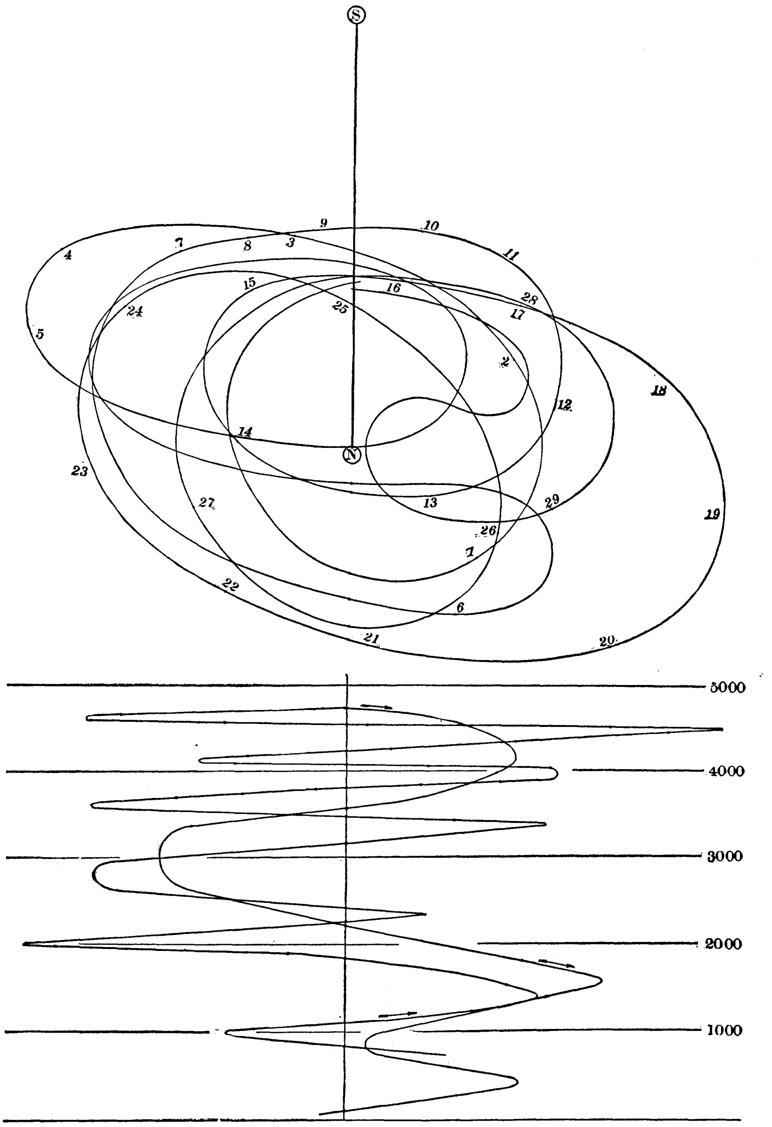


Figure 5. Spiral of Brookins, September 10, plotted from observations of Table III.

## TABLE IV.

FLIGHT OF BROOKINS, SEPT. 8, 1910, WRIGHT BIPLANE. BASE LINE, 6236 FEET. COURSE 13 FEET ABOVE MEAN LOW WATER.

| | Quincy Station. Height above Course 128 ft. | | | | Milton Station. Height above Course 71 ft. | | | |
|---|---|---|---|---|---|---|---|---|
| No. | Time. | Vert. Angle. | Hor. Angle. | Altitude. | Time. | Vert. Angle. | Hor. Angle. | Altitude. |
| 1 | $5^h\ 38^m\ 41^s$ | 2° 29′ | 73° 2′ | 774 ft. | $5^h\ 38^m\ 41^s$ | 2° 48′ | 82° 26′ | 774 ft. |
| 2 | 39 55 | 3 42 | 76 25 | 973 | 39 54 | [3 55] | 76 0 | 967[1] |
| 3 | 41 1 | 4 21 | 84 35 | 1156 | 41 1 | 4 19 | 69 46 | 1154 |
| 4 | 42 9 | 4 13 | 75 17 | 1319 | 42 12 | 4 32 | 82 14 | 1322 |
| 5 | 43 37 | 5 23 | 77 49 | 1506 | 43 36 | 5 36 | 77 34 | 1506 |
| 6 | 48 2 | 5 26 | 73 30 | 1499 | 48 0 | 5 54 | 81 11 | 1516[2] |
| 7 | 49 44 | 7 11 | 81 2 | 1732 | 49 46 | 7 8 | 71 19 | 1732 |
| 8 | 50 59 | 8 31 | 76 4 | 1888 | 51 1 | 8 40 | 73 22 | 1885 |
| 9 | [52] 13 | 5 48 | 80 35 | 1505[3] | 52 7 | 5 52 | 73 17 | 1506 |
| 10 | 54 28 | 6 10 | 80 53 | 1782 | 54 27 | 6 16 | 75 51 | 1783 |
| 11 | 55 35 | 7 3 | 77 3 | 1880 | 55 35 | 7 17 | 77 30 | 1879 |
| 12 | 56 28 | 8 1 | 78 7 | 1959 | 56 26 | ..... | 74 22 | ..... |
| 13 | 57 19 | 9 22 | 79 26 | 2068 | 57 16 | 9 15 | 70 34 | 2068 |
| 14 | 58 8 | 10 41 | 76 10 | 2148 | 58 6 | 10 39 | 70 32 | 2145 |
| 15 | 59 35 | 8 56 | 76 18 | 2254 | 59 35 | 9 12 | 77 13 | 2255 |
| 16 | 6 0 23 | 9 17 | 86 3 | 2363 | 6 0 24 | 8 53 | 68 52 | 2365 |
| 17 | 1 32 | 10 36 | 95 34 | 2456 | 1 31 | 9 22 | 58 59 | 2454 |
| 18 | 2 33 | 12 27 | 100 24 | 2528 | 2 31 | 10 21 | 52 31 | 2532 |
| 19 | 5 11 | [17 42] | 86 51 | 2766[4] | 5 6 | 14 56 | 54 59 | 2759 |
| 20 | 6 28 | [16 38] | 69 35 | 2886[4] | 6 27 | 17 4 | 70 47 | 2884 |
| 21 | 7 36 | 14 31 | 60 20 | 2992 | 7 35 | 16 52 | 85 28 | 2994 |
| 22 | 8 51 | 12 10 | 55 16 | 3102 | 8 51 | 14 50 | 98 9 | 3104 |
| 23 | 9 41 | 11 12 | 62 2 | 3130 | 9 41 | ..... | 93 44 | ..... |
| 24 | 11 56 | 13 35 | 87 15 | 3237 | 11 54 | 12 44 | [66 23] | 3240[5] |
| 25 | 12 56 | 14 46 | 98 4 | 3237 | 12 55 | 12 42 | 55 57 | 3247 |
| 26 | 16 21 | 22 19 | 125 42 | 3419 | 16 16 | 14 44 | 30 49 | 3413 |
| 27 | 17 33 | 26 23 | 111 3 | 3598 | 17 33 | ..... | 36 44 | ..... |
| 28 | 18 46 | 23 32 | 90 3 | 3565 | 18 46 | 19 5 | 51 39 | 3552 |
| 29 | 20 14 | 18 17 | 75 54 | 3647 | 20 4 | 18 6 | 70 35 | 3651 |
| 30 | 22 36 | 14 52 | 86 33 | 3708 | 22 36 | 14 6 | 68 3 | 3715 |
| 31 | 23 45 | 15 21 | 78 25 | 3791 | 23 46 | 15 23 | 74 47 | 3799 |
| 32 | 24 51 | 15 28 | 71 28 | 3831 | 24 51 | 16 21 | 81 7 | 3837 |
| 33 | 26 23 | 12 59 | 91 41 | 3170 | 26 26 | 11 51 | 63 20 | 3168 |

[1] Vert. angle has been increased 1° for Station M.

[2] Marked worthless by observer at M. Clouds between 5 and 6.

[3] Time at Station Q has been decreased by $1^m$.

[4] 5° has been added to each of the vertical angles.

[5] 10° has been added to Hor. Angle at Station M (spiral suggested this change and sextant also at $6^h\ 12^m\ 0^s$ confirms it).

tected in the whole series; it was not indicated by comparison with the Quincy values, as there is close agreement with either value of the horizontal angle, but the spiral showed that the horizontal angle must be in error by about 10°, and a sextant observation at almost the same instant corroborates the value of the height given in the table.

## TABLE V.

FLIGHT OF GRAHAME-WHITE, SEPT. 12, 1910. FARMAN BIPLANE, BASE LINE 6236 FEET, COURSE 13 FEET ABOVE MEAN LOW WATER.

| | Quincy Station.<br>Height above Course 128 ft. | | | | Milton Station.<br>Height above Course 71 ft. | | | |
|---|---|---|---|---|---|---|---|---|
| No. | Time. | Vert. Angle. | Hor. Angle. | Altitude. | Time. | Vert. Angle. | Hor. Angle. | Altitude. |
| 1 | $5^h$ $13^m$ $15^s$ | 2° 25′ | 79° 51′ | 792 ft. | $5^h$ $13^m$ $15^s$ | 2° 37′ | 77° 23′ | 796 ft. |
| 2 | 5 16 12 | 4 17 | 64 6 | 1114 | 5 16 13 | 5 2 | 87 39 | 1115 |
| 3 | 5 16 52 | 4 51 | 70 36 | 1163 | 5 16 51 | 5 21 | 79 14 | 1167 |
| 4 | 5 17 44 | 4 42 | 83 22 | 1206 | 5 17 42 | 4 42 | 70 4 | 1210 |
| 5 | 5 18 30 | 4 11 | 86 3 | 1301 | 5 18 30 | 4 12 | 72 13 | 1307 |
| 6 | 5 19 24 | 3 59 | 78 23 | 1336 | 5 19 18 | 4 13 | 80 50 | 1340 |
| 7 | 5 20 19 | 4 22 | 70 23 | 1369 | 5 20 18 | [4][1] 51 | 87 5 | 1372 |
| 8 | 5 21 2 | 4 51 | 65 34 | 1501 | 5 21 2 | 5 38 | 91 47 | 1525 |
| 9 | 5 21 43 | 5 27 | 62 51 | 1538 | 5 21 44 | 6 22 | 92 13 | 1540 |
| 10 | 5 22 22 | 6 13 | 61 14 | 1582 | 5 22 21 | 7 21 | 90 55 | 1581 |
| 11 | 5 23 10 | 7 14 | 66 6 | 1673 | 5 23 9 | 8 9 | 81 49 | 1682 |
| 12 | 5 23 55 | 8 4 | 78 33 | 1687 | 5 23 54 | 8 1 | [69][2]24 | 1693 |
| 13 | 5 25 23 | 6 13 | 86 14 | 1813 | 5 25 21 | 6 6 | 71 19 | 1811 |
| 14 | 5 26 20 | 6 5 | 77 20 | 1873 | 5 26 19 | 6 21 | 80 36 | 1873 |
| 15 | 5 26 52 | 5 56 | 72 6 | 1854 | 5 26 50 | 6 26 | 85 54 | 1857 |
| 16 | 5 27 28 | 6 9 | 67 14 | 1851 | 5 27 25 | 6 58 | 89 49 | 1873 |
| 17 | 5 28 3 | 6 40 | 63 29 | 1910 | 5 28 2 | 7 40 | 92 24 | 1909 |
| 18 | 5 28 55 | 8 0 | 63 51 | 1981 | 5 28 55 | 9 10 | 87 56 | 1981 |
| 19 | 5 29 31 | 8 54 | 71 59 | 2020 | 5 29 30 | 9 29 | 77 44 | 2029 |
| 20 | 5 30 17 | 8 32 | 83 49 | 2111 | 5 30 18 | 8 20 | 69 53 | 2121 |
| 21 | 5 31 8 | 7 17 | 78 35 | 2104 | 5 31 8 | 7 31 | 78 10 | 2114 |
| 22 | 5 31 43 | 7 12 | 72 23 | 2093 | . . . . . . | 7 42 | 84 7 | 2086 |
| 23 | 5 32 20 | 6 7 | 67 47 | 1687 | 5 32 20 | 6 50 | 86 52 | 1687 |
| 24 | 5 33 14 | 3 27 | 66 5 | 859 | 5 33 14 | 4 0 | 83 12 | 851 |

[1] Vert. angle at Milton recorded 5° 51′.

[2] Hor. angle recorded at Milton was 79° 24′. Spiral proves it to be 69° 24′ and sextant observation at almost same instant corroborates this.

It has been thought worth while to give these observations in detail for those especially interested in the problem. It is evident that there is much information in the tables and curves to which we have not referred; for example, rate of ascent per minute, gradient with and against the wind, etc. The reader who wishes to study them more minutely may therefore wish to plot them on a scale sufficiently large for his purpose. To make use of the sextant as well as the transit observations in plotting the spirals, it is necessary to know the relative position of the two bases. The north end of the field base is distant 15,000 feet and 14,700 feet from Q and M respectively. The south end of the field base is 10,056 feet from Q and 11,005 feet from M.

In regard to these tables we notice that there is apparently a small systematic difference between heights as observed at Q and M. This was at first suspected to be due to error in level of the two stations, but was not the same at all times. It was not due to difference in time of observation at the two stations, for although the setting at Milton was always a fraction of a second later, this could only give rise to a difference of about one foot at most. It may have been partly due to difference in levelling of instruments, and perhaps partly to neglect of refraction and curvature of the earth in the computation.

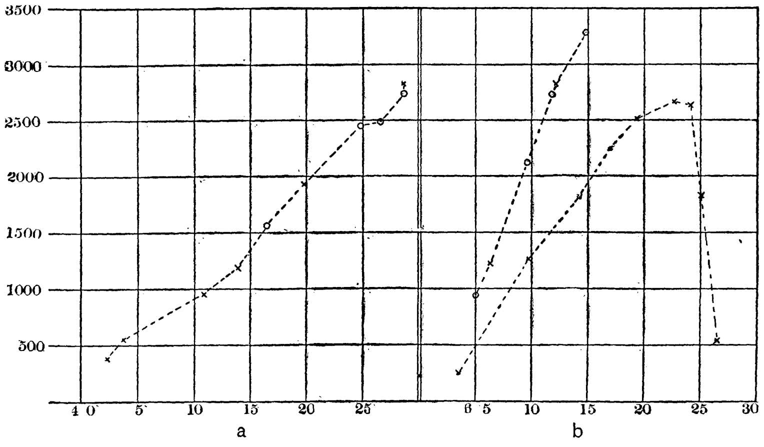


Figure 6. Time-altitude curve. (Left) flight of Johnstone, September 7 (Wright biplane); (right) flight of Grahame-White (Blériot monoplane), and flight of Brookins (Wright biplane).

The time-altitude curves have been plotted for all the flights of the meet which have any special interest. Those of Brookins, Johnstone, and Grahame-White on September 7 are plotted in Figure 6. They show the typical rate of ascent of the Wright biplane, about 100 feet per minute, in direct comparison with the Blériot which was in the air at the same time and rising more than 200 feet per minute.

Figure 4, plotted from Table III, represents the highest flight of the meet, that of Brookins on September 10, at an average rate of about 100 feet per minute. From $5^h\ 43^m$ to $5^h\ 57^m$ the transits were temporarily assigned to Grahame-White in a Farman biplane, who was far off the course to the southeast and could not be frequently observed by the sextants, while the latter instruments were sufficient for the observations of Brookins, who was describing large but regular circles above the field. At $5^h\ 52^m$ Grahame-White was cut off from view at Milton by a large tree which lay farther off the course than it was supposed that any aviator would go in an altitude attempt, and the transits were turned back upon Brookins.

The single sextant observation of Grahame-White's descent, however, combined with the fact that his usual rate on the beginning of the downward spiral was at least 800 feet per minute, enables us to fill out the dotted part of the curve, and conclude that he reached a height of approximately 2200 feet. A comparison of all the curves shows that if the highest point of the ascent is not actually observed by the transits, it may always be closely fixed by the intersection of the upper portions of the curves of ascent and descent.

The spiral of Brookins in this flight is shown in Figure 5. In the long interval between points 5 and 6, when the transits were turned upon Grahame-White, the general curve of the spiral, however, is plainly shown by the sextant observations, although the aviator changed the direction in which he was circling the course, apparently in order to make his crossings against the wind.

Figure 7 shows the time-altitude curve of Brookins on September 8, when he was lost to sight in a light cloud for five minutes at an altitude of 1500 feet, climbed another 500 feet and disappeared a second time, when he stopped his engine, descending to the 1500 foot level again out of the cloud, and then taking a somewhat different line of flight, ascended without further difficulties to a height of nearly 4000 feet. The conditions were such that the early part of the spiral was not well determined, and it does not seem worth while to reproduce it here. The main features are shown on the ground plan which is shown on the map of Plate 1. The very discordant sextant observations at $5^h\ 19^m$ and $5^h\ 22^m$, shown in Figure 7, are unexplained. The former was made with the machine about 1000 feet south of the southern station, nearly overhead, and moving in a course nearly parallel to the line, so that the vertical angle was changing very rapidly, and the observation difficult.

No such cause can be assigned for the second error, and it is perhaps more probable that both were caused by some accidental disturbance of the vertical plane of the sextant which affected the two observations alike. No other discrepancy approaching this in amount occurred in in the whole course of the observations.

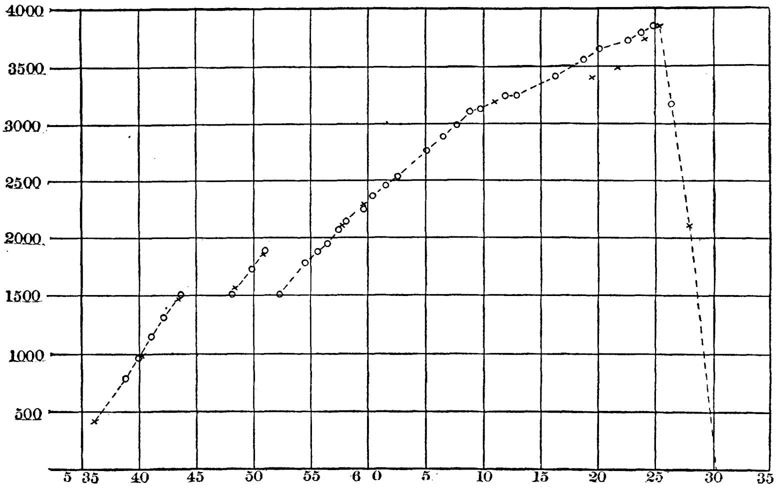
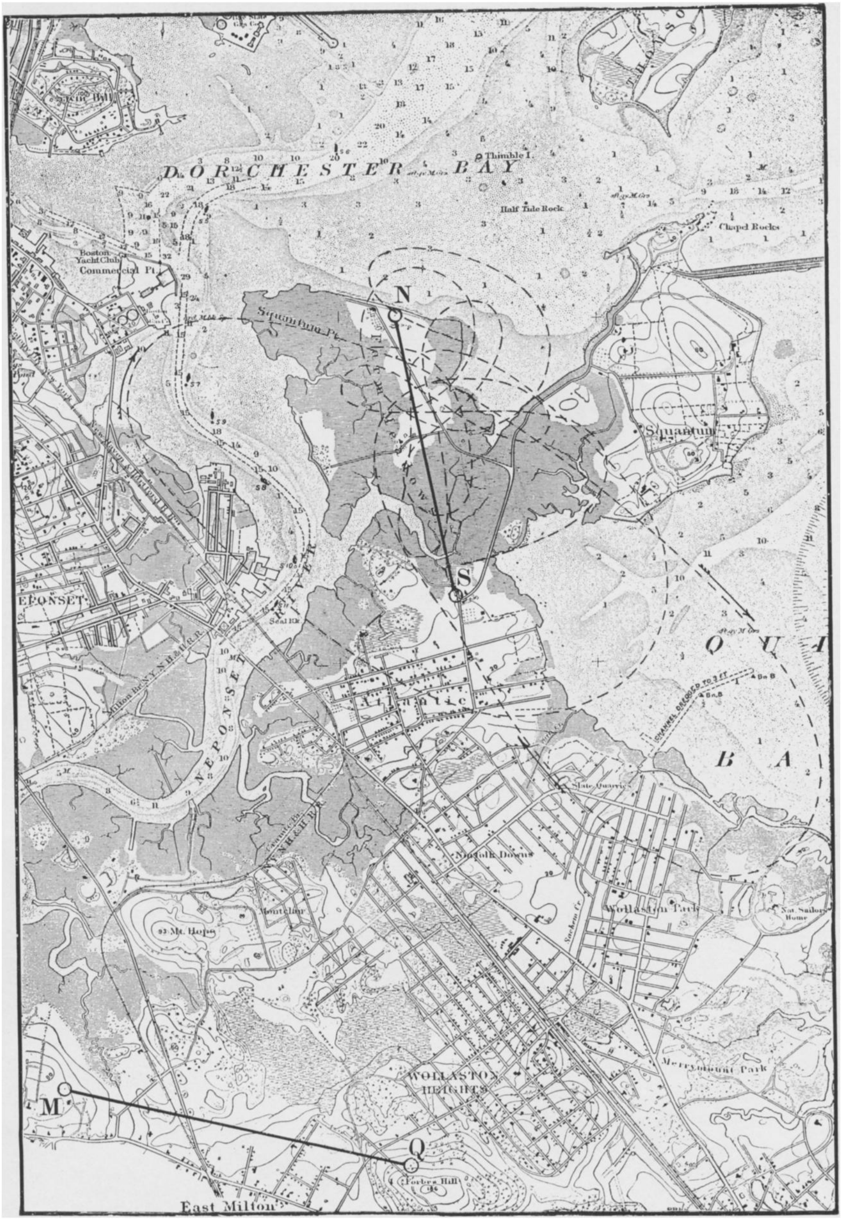


Figure 7. Time-altitude curve: Brookins (Wright biplane), September 8.

It remains to make grateful acknowledgment of the skill of the observers and of the interest of many others, without which this work would have been impossible; especially of the assistance of Mr. William Hunt in the observations and reductions, and in drawing the curves which illustrate this paper.

Students' Astronomical Laboratory,
Harvard University.

PLATE I.


DORCHESTER BAY
Thimble I.
Half Tide Rock
Chapel Rocks
Boston Yacht Club
Commercial Pt.
Squantum Pt.
N
S
Squantum
NEPONSET
NEPONSET RIVER
Seal Rk.
Atlantic
QUINCY BAY
State Quarries
Norfolk Downs
Wollaston Park
Nat. Sailors Home
Montclair
93 Mt. Hope
Merrymount Park
WOLLASTON HEIGHTS
M
Q
Forbes Hill
East Milton